Logic, Information flow and Argumentation

Homework exercises, Week 2, part a (due Friday 17. February).

1. On Slides 4 and 5, you see how the *truth value* of each logical connective is defined (by the *truth tables*). To evaluate the truth of a complex formula containing more than one connective, you follow the construction tree of the formula to compute the truth values bottom up - you can see this on Slide 8. For example, suppose you want to evaluate the truth value of the expression "$(p \lor q) \land (r \lor s)$", assuming that p is true and q, r, s are false (i.e., the *valuation* V defined by $V(p) = 1$, $V(q) = 0$, $V(r) = 0$, $V(s) = 0$). Then proceed as follows:

	$(p$	$\lor$	$q)$	$\land$	$(r$	$\lor$	$s)$
	1		0		0		0
step 1:	1	**1**	0		0	**0**	0
step 2:	1	**1**	0	**0**	0	**0**	0

So the truth value of the entire expression is 0 under the given truth value of p, q, r, s (i.e., under the given valuation V).

Evaluate the truth of the following expressions in propositional logic, given the truth values of the atomic propositions:

   (a) $(\neg p \lor q)$, assuming that p and q are both true,

   (b) $p \lor (q \land (\neg p))$, assuming that p is false and q is true

   (c) $p \rightarrow (q \land (\neg q))$, assuming that p and q are both false

   (d) $(p \land q) \leftrightarrow (\neg r)$, assuming that p is true and q, r are false,

   (e) $(p \land q) \lor (r \land s)$, assuming that p, q are true and r, s are false,

   (f) $p \rightarrow (q \rightarrow (r \rightarrow s))$, assuming that p, q, r are true and s is false,

   (g) $\neg(p \rightarrow q) \lor (r \rightarrow s)$, assuming that p, q, r are true and s is false,

(h) $((p \rightarrow (q \vee r)) \rightarrow s) \wedge ((p \wedge q) \vee (r \rightarrow s))$, assuming that p, q, r, s are all false.

2. You can evaluate the truth of an expression in all possible situations (i.e., for all possible *valuations*, see Slide 9). For example, if you have two propositional variable p and q and want to evaluate the expression "$(p \vee q) \rightarrow p$", you do the same as in Exercise 1, for four different combinations of truth-value assignments to p and q (i.e., four different valuations):

(p	$\vee$	q)	$\rightarrow$	p
1	1	1	1	1
1	1	0	1	1
0	1	1	0	0
0	0	0	1	0

Evaluate the truth of the following expressions in all possible situations (for all possible valuations):

(a) $p \vee (q \vee r)$,

(b) $((p \rightarrow q) \rightarrow p) \rightarrow p$

(c) $(p \wedge \neg p) \vee (p \rightarrow p)$,

(d) $\neg(p \vee q) \leftrightarrow ((\neg p) \wedge (\neg q))$,