Amsterdam University College

Logic, Information flow and Argumentation

Homework exercises, Week 4, part a (due Friday 2 March).

1. By means of natural deduction, prove the following (conjunction only):

    (a) from premise $p \wedge q$ derive conclusion p

    (b) from premise $(p \wedge q) \wedge r$ derive conclusion r

    (c) from premise $(p \wedge q) \wedge r$ derive conclusion p

    (d) from premises p and q derive conclusion $p \wedge q$

    (e) from premise $p \wedge q$ derive conclusion $q \wedge p$

    (f) from premises p and $q \wedge r$ derive conclusion $p \wedge q$

2. By means of natural deduction, prove the following (adding disjunction):

    (a) from premise r derive conclusion $p \vee r$

    (b) from premise q derive conclusion $(r \wedge p) \vee q$

    (c) from premise $p \wedge q$ derive $q \vee r$

    (d) from premise $p \wedge (r \wedge q)$ derive conclusion $r \vee (p \wedge q)$

    (e) from premises $(p \vee q) \wedge r$ and $r \vee q$ derive $p \vee q$

    (f) from premise $(p \wedge q) \vee p$ derive conclusion p

    (g) from premise $p \vee q$ derive conclusion $q \vee p$

    (h) from premise $(p \wedge q) \vee (r \wedge q)$ derive conclusion q

    (i) from premise $(p \wedge q) \vee (r \wedge q)$ derive conclusion $q \vee (s \wedge t)$

3. By means of natural deduction, prove the following (adding implication):

    (a) derive $(p \wedge q) \rightarrow q$ (N.B. no premises given!)

    (b) derive $(p \wedge q) \rightarrow (r \vee q)$

    (c) derive $(p \vee q) \rightarrow (q \vee p)$

    (d) from premises $p \wedge q$ and $q \rightarrow r$ derive conclusion r

    (e) from premises r and $(p \vee r) \rightarrow q$ derive conclusion q

(f) from premises $p \wedge r$ and s derive $(p \vee r) \rightarrow s$

(g) from premises $p \wedge q$ and s derive $(q \vee r) \rightarrow s$

(h) from premises $p \vee (r \wedge s)$ and s derive $(p \vee s) \rightarrow s$

(i) from premises r and $r \rightarrow s$ derive s

(j) from premises $r \wedge s$ and $s \rightarrow t$ derive t

(k) from premises $p \rightarrow q$ and p derive $q \vee r$

(l) from premises $(p \wedge q) \vee (r \wedge q)$ and $q \rightarrow s$ derive s

(m) from premises $(p \vee q) \rightarrow r$ and $r \wedge q$ derive r

4. By means of natural deduction, prove the following (adding negation):

(a) derive $\neg(p \wedge \neg p)$ (N.B. no premises given!)

(b) derive $p \rightarrow p$

(c) derive $p \rightarrow (p \vee q)$

(d) from premise $\neg p$ derive $\neg(p \wedge q)$

(e) from premises $p \wedge q$ and $\neg(p \wedge q)$ derive $\neg r$

(f) from premises $p \rightarrow q$ and $\neg q$ derive $\neg p$