Logic in Action Chapter 2: Propositional Logic

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In a restaurant, your Father has ordered Fish, your Mother ordered Vegetarian, and you ordered Meat. Out of the kitchen comes some new person carrying the three plates. What will happen?

Three guests are sitting at a table. The waitress asks: "Does everyone want coffee". The first guest says: "I don't know". The second guest now says: "I don't know". Then the third guest says: "No, not everyone wants coffee". The waitress comes back and gives the right people their coffees. Assuming that at the beginning each guest only knows about himself, which was the waitress reasoning? Who gets coffee and who does not?

Example (3)



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Example (3)



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Example (3)



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Example (3)

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Example (3)

1	2	3
3	1	2
2	3	1

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If you take the medication, you will get better. You are taking the medication.

So, you will get better.

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So, you took the medication.

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So, you took the medication.

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If you take the medication, you will get better. But you are not taking the medication.

So, you will not get better.

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If you take the medication, you will get better. But you are not taking the medication.

So, you will not get better.

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If you take the medication, you will get better. But you are not taking the medication.

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If you take the medication, you will get better. But you are not getting better.

So, you have not taken the medication.

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Valid inference

 $egin{array}{c} A_1,\ldots,A_n \\ \hline C \end{array}$

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Valid inference

 A_1,\ldots,A_n \boldsymbol{C}

An inference is **valid** if and only if **every** time **all** the premises are true, the conclusion is also true.

What a valid inference tells us?

Suppose the following inference is valid

$egin{array}{c} A_1,\ldots,A_n \\ \hline C \end{array}$

Then

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What a valid inference tells us?

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Then

• if all the premises A_1, \ldots, A_n are true, so is the conclusion C.

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What a valid inference tells us?

Suppose the following inference is valid

 $egin{array}{c} A_1,\ldots,A_n \ \hline C \end{array}$

Then

- **()** if all the premises A_1, \ldots, A_n are true, so is the conclusion C.
- **2** if the conclusion C is false, at least one premise A_i is false.

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Two valid inferences:

If you take the medication, then you will get better. You are taking the medication.

So, you will get better.

If you jump from a 4th floor, then you will fly. You jump from a 4th floor.

So, you will fly.

Two valid inferences:

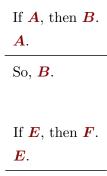
If you take the medication, then you will get better. You are taking the medication.

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Two valid inferences:



So, \boldsymbol{F} .

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Another valid inference:

If you take the medication, then you will get better. You are not getting better.

So, you are not taking the medication.

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Another valid inference:

If you take the medication, then you will get better. You are not getting better.

So, you are not taking the medication.

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Validity and Inference Patterns

Looking for patterns (2)

Another valid inference:

If \boldsymbol{A} , then \boldsymbol{B} . not \boldsymbol{B} .

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So, not **A**.

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And yet another:

An integer x is even or odd. If x is even, then x + x is even. If x is odd, then x + x is even.

So, x + x is even.

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And yet another:

An integer x is **even** or **odd**. If x is **even**, then x + x is **even**. If x is **odd**, then x + x is **even**.

So, x + x is even.

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And yet another:

An integer x is A_1 or A_2 . If x is A_1 , then C. If x is A_2 , then C.

So, *C*.

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And yet another:

An integer x is A_1 or A_2 . If x is A_1 , then C. If x is A_2 , then C.

So, *C*.

Can you think of others?

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Validity and Inference Patterns

The main question

How can we recognize valid inference patterns?

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- A1 At least one of them is guilty.
- A2 Not all of them are guilty.
- A3 If Mrs White is guilty, then Colonel Mustard helped her (he is guilty too).
- A4 If Miss Scarlet is innocent then so is Colonel Mustard.

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 A_1, A_2, A_3, A_4 ?

Miss Scarlet is guilty

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 A_1, A_2, A_3, A_4 1

Miss Scarlet is guilty

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 $\checkmark \quad \underline{\qquad \qquad A_1,A_2,A_3,A_4}$

Miss Scarlet is guilty

In every situation in which A_1, A_2, A_3 and A_4 are all true, "Miss Scarlet is guilty" is true.

 $\checkmark \quad \underline{\qquad \qquad A_1,A_2,A_3,A_4}$

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$$A_1, A_2, A_3, A_4$$

Mrs White is innocent

 $\checkmark \quad \underbrace{A_1,A_2,A_3,A_4}$

Miss Scarlet is guilty

In every situation in which A_1, A_2, A_3 and A_4 are all true, "Miss Scarlet is guilty" is true.

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Mrs White is innocent

 $\checkmark \quad \frac{A_1, A_2, A_3, A_4}{\text{Miss Scarlet is guilty}}$

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$$\checkmark \quad \underline{A_1, A_2, A_3, A_4}$$

Mrs White is innocent

In every situation in which A_1, A_2, A_3 and A_4 are all true, "Mrs White is innocent" is true.

 A_1, A_2, A_3, A_4 ?

Colonel Mustard is guilty

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Colonel Mustard is guilty

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There is a situation in which A_1, A_2, A_3 and A_4 are all true, but "Colonel Mustard is guilty" is false (there is a counter-example).

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Colonel Mustard is innocent

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Colonel Mustard is innocent

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The Language of Propositional Logic

Ingredients of the **propositional** language

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Ingredients of the **propositional** language

• Basic (*atomic*) statements (**propositions**):

 $\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \dots$

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• Basic (*atomic*) statements (**propositions**):

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2 Operators to build more statements:

" not "	becomes	٦
" and "	becomes	
" or "	becomes	V
" if then "	becomes	$\ldots \rightarrow \ldots$
" if and only if"	becomes	$\ldots \leftrightarrow \ldots$

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The language \mathcal{L}_P is a set of formulas satisfying:

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The language \mathcal{L}_P is a set of formulas satisfying:

• All the basic propositions are in \mathcal{L}_{P} :

$$oldsymbol{p}\in\mathcal{L}_{ extsf{P}}, \quad oldsymbol{q}\in\mathcal{L}_{ extsf{P}}, \quad oldsymbol{r}\in\mathcal{L}_{ extsf{P}}, \quad \ldots$$

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 and $\psi \in \mathcal{L}_{\mathsf{P}}$, then
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The language \mathcal{L}_P is a set of formulas satisfying:

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2) If
$$\varphi \in \mathcal{L}_{P}$$
 and $\psi \in \mathcal{L}_{P}$, then
 $\neg \varphi \in \mathcal{L}_{P}$, $(\varphi \land \psi) \in \mathcal{L}_{P}$,

3

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$$\begin{array}{l} \textbf{@} \mbox{ If } \boldsymbol{\varphi} \in \mathcal{L}_{\mathsf{P}} \mbox{ and } \boldsymbol{\psi} \in \mathcal{L}_{\mathsf{P}}, \mbox{ then} \\ \neg \boldsymbol{\varphi} \in \mathcal{L}_{\mathsf{P}}, \qquad (\boldsymbol{\varphi} \wedge \boldsymbol{\psi}) \in \mathcal{L}_{\mathsf{P}}, \\ (\boldsymbol{\varphi} \lor \boldsymbol{\psi}) \in \mathcal{L}_{\mathsf{P}}, \end{array}$$

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3 Nothing else is in \mathcal{L}_{P} .

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3 Nothing else is in \mathcal{L}_{P} .

In practice, we will avoid parenthesis if they are not necessary.

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The construction of a formula can be seen as building a **tree**.

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 $((\neg p \lor q) \to r)$

The construction of a formula can be seen as building a **tree**.

$$\begin{array}{c} ((\neg p \lor q) \to r) \\ \downarrow \\ \to \end{array}$$

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The construction of a formula can be seen as building a **tree**.

$$((\neg p \lor q) \rightarrow r)$$
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$$((\neg p \lor q) \rightarrow r)$$

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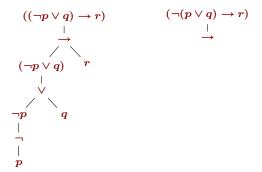
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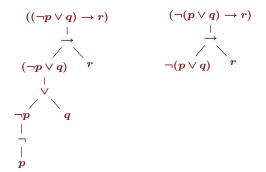
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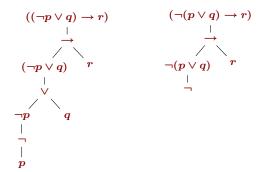
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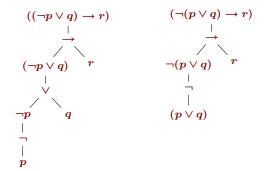
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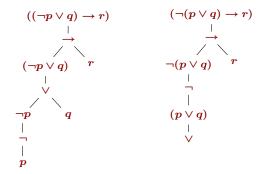
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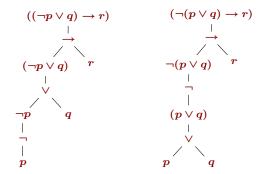
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Evaluating formulas

How do we know if a given formula φ is **true** or **false**?

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How do we know if a given formula φ is **true** or **false**?

• We need the **truth-values** of the basic propositions p, q, r, ... that appear in φ .

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Evaluating formulas

How do we know if a given formula φ is **true** or **false**?

- We need the **truth-values** of the basic propositions p, q, r, ... that appear in φ .
- We need to know the **meaning** of \neg , \land , \lor , \rightarrow and \leftrightarrow .

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Behaviour of the connectives (1)

Use 1 for **true**, and 0 for **false**.

For negation \neg



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arphi	$\neg \varphi$
1	0
0	

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Behaviour of the connectives (1)

Use 1 for **true**, and 0 for **false**.

For negation \neg

arphi	$\neg \varphi$
1	0
0	1

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Use 1 for true, and 0 for false. For negation \neg

arphi	$\neg \varphi$
1	0
0	1

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or, in a shorter format:

arphi

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Use 1 for true, and 0 for false. For negation \neg

arphi	$\neg arphi$
1	0
0	1

or, in a shorter format:

-	arphi	
	1	
	0	

Use 1 for true, and 0 for false. For negation \neg

arphi	$\neg arphi$
1	0
0	1

or, in a shorter format:

7	arphi
0	1
	0

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Use 1 for true, and 0 for false. For negation \neg

arphi	$\neg \varphi$	
1	0	
0	1	

or, in a shorter format:

	arphi
0	1
1	0

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Behaviour of the connectives (2)

For conjunction \land

 \wedge ψ φ

(http://www.logicinaction.org/)

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Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1		1
1		0
0		1
0		0

(http://www.logicinaction.org/)

Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	ψ
1	1	1
1		0
0		1
0		0

(http://www.logicinaction.org/)

Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	ψ
1	1	1
1	0	0
0		1
0		0

(http://www.logicinaction.org/)

Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0		0

(http://www.logicinaction.org/)

Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0

(http://www.logicinaction.org/)

Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0

For **disjunction** \lor

arphi arphi ψ

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Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0
arphi	\vee	$oldsymbol{\psi}$
1		1
1		0
0		1
0		0

For **disjunction** \lor

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Behaviour of the connectives (2)

For conjunction \wedge

For disjunction \vee

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0
arphi	\vee	$oldsymbol{\psi}$
1	1	1
1		0

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0

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Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0
arphi	\vee	$oldsymbol{\psi}$
1	1	1

1 **1**

0

0

0

0

For **disjunction** \lor

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Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0
arphi	\vee	$oldsymbol{\psi}$
1	1	1
1	1	0

0 1 1

0

0

For **disjunction** \lor

Behaviour of the connectives (2)

For conjunction \wedge

arphi	\wedge	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	0	0
arphi	\vee	$oldsymbol{\psi}$
1	1	1

For **disjunction** \lor

arphi	\vee	ψ
1	1	1
1	1	0
0	1	1
0	0	0

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Behaviour of the connectives (3)

For equivalence \leftrightarrow

$$arphi ~\leftrightarrow~ \psi$$

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Behaviour of the connectives (3)

For equivalence \leftrightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1		1
1		0
0		1
0		0

(http://www.logicinaction.org/)

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Behaviour of the connectives (3)

For equivalence \leftrightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1		0
0		1
0		0

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Semantic Situations: Truth Tables

Behaviour of the connectives (3)

For equivalence \leftrightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0		1
0		0

(http://www.logicinaction.org/)

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Semantic Situations: Truth Tables

Behaviour of the connectives (3)

For equivalence \leftrightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0		0

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Semantic Situations: Truth Tables

Behaviour of the connectives (3)

For equivalence \leftrightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	1	0

(http://www.logicinaction.org/)

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For equivalence \leftrightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	1	0

For implication \rightarrow

 $arphi ~
ightarrow ~\psi$

For equivalence \leftrightarrow

	arphi	\leftrightarrow	$oldsymbol{\psi}$
	1	1	1
	1	0	0
	0	0	1
	0	1	0
For implication \rightarrow			
	arphi	\rightarrow	ψ
	1		1
	1		0
	0		1
	0		0
	0		0

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For equivalence \leftrightarrow

For **implication** \rightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	1	0
arphi	\rightarrow	ψ
φ 1	\rightarrow 1	ψ 1
1		1

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For equivalence \leftrightarrow

For **implication** \rightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	1	0
-		
φ	\rightarrow	ψ
	\rightarrow 1	ψ 1
φ		
φ 1	1	1

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For equivalence \leftrightarrow

For **implication** \rightarrow

arphi	\leftrightarrow	$oldsymbol{\psi}$
1	1	1
1	0	0
0	0	1
0	1	0
	-	
 φ	\rightarrow	ψ
φ	\rightarrow	ψ
φ 1	\rightarrow 1	ψ 1

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For equivalence \leftrightarrow

For **implication** \rightarrow

$\begin{array}{cccccccccccccccccccccccccccccccccccc$			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	arphi	\leftrightarrow	$oldsymbol{\psi}$
$\begin{array}{cccc} 0 & 0 & 1 \\ 0 & 1 & 0 \end{array}$ $\begin{array}{c} \varphi & \longrightarrow & \psi \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}$	1	1	1
$\begin{array}{cccc} 0 & 1 & 0 \\ \hline \psi \\ \hline \psi \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}$	1	0	0
$\begin{array}{ccc} \varphi & \rightarrow & \psi \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}$	0	0	1
1 1 1 1 0 0 0 1 1	0	1	0
$\begin{array}{cccc} 1 & {\bf 0} & 0 \\ 0 & {\bf 1} & 1 \end{array}$			
0 1 1	φ	\rightarrow	ψ
0 1 0	1	1	1
	1 1	1 0	$\begin{array}{c} 1\\ 0 \end{array}$

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Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

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Example: assume $P = \{p, q\}$.

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Example: assume $P = \{p, q\}$. There are **four** different valuations (**four** different situations):

Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

Example: assume $P = \{p, q\}$. There are **four** different valuations (**four** different situations):

 $V_1(\boldsymbol{p}) = 1 \quad V_1(\boldsymbol{q}) = 1$

Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

Example: assume $P = \{p, q\}$. There are **four** different valuations (**four** different situations):

$$V_1(p) = 1$$
 $V_1(q) = 1$
 $V_2(p) = 1$ $V_2(q) = 0$

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Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

Example: assume $P = \{p, q\}$. There are **four** different valuations (**four** different situations):

$$V_1(p) = 1 V_1(q) = 1$$

$$V_2(p) = 1 V_2(q) = 0$$

$$V_3(p) = 0 V_3(q) = 1$$

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Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

Example: assume $P = \{p, q\}$. There are **four** different valuations (**four** different situations):

$$V_1(p) = 1 V_1(q) = 1$$

$$V_2(p) = 1 V_2(q) = 0$$

$$V_3(p) = 0 V_3(q) = 1$$

$$V_4(p) = 0 V_4(q) = 0$$

Valuation. Let $P = \{p, q, r, ...\}$ be a set of atomic propositions. A valuation V from P to $\{0, 1\}$ assigns to each element of P a unique truth-value.

Example: assume $P = \{p, q\}$. There are **four** different valuations (**four** different situations):

$$V_1(p) = 1 V_1(q) = 1$$

$$V_2(p) = 1 V_2(q) = 0$$

$$V_3(p) = 0 V_3(q) = 1$$

$$V_4(p) = 0 V_4(q) = 0$$

How many for $P = \{p\}$? How many for $P = \{p, q, r\}$?

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$$(\neg p) \land q$$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

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$$\begin{array}{cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

 $(p \land (p \rightarrow q)) \rightarrow q$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

 $V: 1 \qquad 1 \qquad 0 \qquad 0$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

V: 1 1 0 0 0

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

 $V: 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$\begin{array}{ccccc} (p & \wedge & (p & \rightarrow & q)) & \rightarrow & q \\ V: & 1 & 0 & 1 & 0 & 0 & \mathbf{1} & 0 \end{array}$$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

 $V: 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0$
 $V \models (p \land (p \rightarrow q)) \rightarrow q$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$



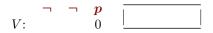
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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

 $V: 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0$
 $V \models (p \land (p \rightarrow q)) \rightarrow q$



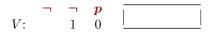
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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

 $V: 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0$
 $V \models (p \land (p \rightarrow q)) \rightarrow q$



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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$



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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$(p \land (p \rightarrow q)) \rightarrow q$$

 $V: 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0$
 $V \models (p \land (p \rightarrow q)) \rightarrow q$

$$V: \begin{array}{ccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$V: \begin{array}{cccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$

$$(p \rightarrow q) \lor (q \rightarrow p)$$

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$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$V: \begin{array}{ccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$

$$(p \rightarrow q) \lor (q \rightarrow p)$$

V: 0 1 1 0

(http://www.logicinaction.org/)

$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$V: \begin{array}{ccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$

$$\begin{array}{cccc} (p & \rightarrow & q) & \lor & (q & \rightarrow & p) \\ V: & 0 & 1 & 1 & & 1 & & 0 \end{array}$$



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Evaluating formulas in one situation

$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$V: \begin{array}{ccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$

$$\begin{array}{cccc} (p & \rightarrow & q) & \lor & (q & \rightarrow & p) \\ V: & 0 & 1 & 1 & & 1 & 0 & 0 \end{array}$$

Evaluating formulas in one situation

$$\begin{array}{c|cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$V: \begin{array}{ccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$



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Evaluating formulas in one situation

$$\begin{array}{cccc} (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 1 \end{array} \quad \boxed{V \models (\neg p) \land q}$$

$$V: \begin{array}{ccc} \neg & \neg & p \\ 0 & 1 & 0 \end{array} \quad \boxed{V \not\models \neg \neg p}$$

$$(p \rightarrow q) \lor (q \rightarrow p)$$
 $V \models (p \rightarrow q) \lor (q \rightarrow p)$ $V \models (p \rightarrow q) \lor (q \rightarrow p)$

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Semantic Situations: Truth Tables

Evaluating formulas in all possible situations

$$(p \land (p \rightarrow q)) \rightarrow q$$

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Semantic Situations: Truth Tables

Evaluating formulas in all possible situations

(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	q
1		1		1		1
1		1		0		0
0		0		1		1
0		0		0		0

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Semantic Situations: Truth Tables

Evaluating formulas in all possible situations

(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	q
1		1	1	1		1
1		1	0	0		0
0		0	1	1		1
0		0	1	0		0

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(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	\boldsymbol{q}
1	1	1	1	1		1
1	0	1	0	0		0
0	0	0	1	1		1
0	0	0	1	0		0

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(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	q
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0

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(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	${m q}$
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0
		_		p		

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(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	q
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0
		-	_	p		
				1		
				0		

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(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	q
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0
		-		p		
			0	1		
			1	0		

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(<i>p</i>	\wedge	(<i>p</i>	\rightarrow	q))	\rightarrow	\boldsymbol{q}
1	1	1	1	1	1	1
1	0	1	0	0	1	0
0	0	0	1	1	1	1
0	0	0	1	0	1	0
		_	-	\boldsymbol{p}		
		1	0	1		
		0	1	0		

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• Those that are never true (contradiction):

 $p \wedge (\neg p), \ldots$

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• Those that are never true (contradiction):

 $p \wedge (\neg p), \ldots$

• Those that can be true (**satisfiable**):

 $(\neg p) \lor q, \dots$

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• Those that are never true (contradiction):

 $p \wedge (\neg p), \ldots$

• Those that can be true (**satisfiable**):

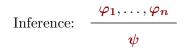
 $(\neg p) \lor q, \dots$

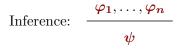
• Those that are always true (valid, tautology):

 $(p \land (p
ightarrow q))
ightarrow q, \ldots$

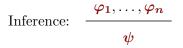
If the formula φ is valid, we write $\models \varphi$

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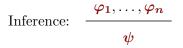


Valid inference. An inference is valid if and only if every time (every situation) in which all premises $\varphi_1, \ldots, \varphi_n$ are true, ψ is also true.



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We also say ψ is a **logical consequence** of $\varphi_1, \ldots, \varphi_n$.

We will write $\varphi_1, \ldots, \varphi_n \models \psi$

Our previous patterns:

$$p \mid (p \rightarrow q) \parallel q$$

(http://www.logicinaction.org/)

Examples

Our previous patterns:

p	(<i>p</i>	\rightarrow	q)	q
1	1	1	$\begin{array}{c} 1 \\ 0 \end{array}$	1
1	$ \begin{array}{c} 1 \\ 1 \\ 0 \\ 0 \end{array} $	0	0	1 0 1 0
0 0	0	1	1	1
0	0	1	0	0

Our previous patterns:

	p	(<i>p</i>	\rightarrow	q)	q
\rightarrow	1	1	1 0 1 1	$\begin{array}{c} 1 \\ 0 \end{array}$	1
	1	1	0	0	0
	0	0	1	$\begin{array}{c} 1 \\ 0 \end{array}$	0 1 0
	0	$\begin{array}{c}1\\1\\0\\0\end{array}$	1	0	0

Our previous patterns:

	$p \mid$	(<i>p</i>	\rightarrow	<i>q</i>)	q
\rightarrow	1	1	1	1	1
	1	1	0	0	0
	1 0 0	0	1	1	1
	0	0	1	0	0
	$\neg q$	(<i>p</i>	\rightarrow	<i>q</i>)	$ \neg p$

(http://www.logicinaction.org/)

Our previous patterns:

	p	(<i>p</i>	\rightarrow	q)	q	
\rightarrow	1	1	1	1	1	
	1	1	0	0	0	
	0	0	1	1	1	
	0	0	1	0	0	
<u> </u>						
	$\neg q$	(<i>p</i>	\rightarrow	<i>q</i>)	$ \neg p$	
	¬q 0	(p	\rightarrow 1	q) 1	$ \neg p$ 0	
	-		\rightarrow 1 0		-	
	0	1		1	0	
	0 1	1 1	0	$\begin{array}{c} 1\\ 0 \end{array}$	0	

(http://www.logicinaction.org/)

Our previous patterns:

	p	(<i>p</i>	\rightarrow	q)	q
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	1	1	0	0	0
	0	0	1	1	1
	0	0	1	0	0
	eg q	(<i>p</i>	\rightarrow	q)	$ \neg p$
	0	1	1	1	0
	1	1	0	0	0
	0	0	1	1	1
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	p	(<i>p</i>	\rightarrow	<i>q</i>)	q
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	1	1	0	0	0
	0	0	1	1	1
	0	0	1	0	0
	eg q	(<i>p</i>	\rightarrow	<i>q</i>)	$ \neg p$
	0	1	1	1	0
	1	1	0	0	0
	0	0	1	1	1
\rightarrow	1	0	1	0	1

What about the others?

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Further definitions

(http://www.logicinaction.org/)

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Further definitions

• Two formulas φ and ψ are logically equivalent ($\varphi \equiv \psi$) if and only if $\varphi \models \psi$ and $\psi \models \varphi$.

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Further definitions

- Two formulas φ and ψ are logically equivalent ($\varphi \equiv \psi$) if and only if $\varphi \models \psi$ and $\psi \models \varphi$.
- A set of formulas **X** is **satisfiable** if and only if **there is one valuation** that makes **every** formula in **X** true.

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Proof

Symbolic inference

(http://www.logicinaction.org/)

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Symbolic inference

• A **proof** is a finite sequence of formulas where each formula is either an *axiom* or else it has been infered from previous formulas by using an **inference rule**.

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\mathbf{Proof}

Symbolic inference

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- A formula is a **theorem** if it occurs in a proof.

3

\mathbf{Proof}

Symbolic inference

- A **proof** is a finite sequence of formulas where each formula is either an *axiom* or else it has been infered from previous formulas by using an **inference rule**.
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- A set of axioms and rules is called an **axiom system** or an **axiomatization**.

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Symbolic inference

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Symbolic inference

- A **proof** is a finite sequence of formulas where each formula is either an *axiom* or else it has been infered from previous formulas by using an **inference rule**.
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- A set of axioms and rules is called an **axiom system** or an **axiomatization**.
 - An axiom system is **sound** for a logic if every theorem is valid in the logic.
 - An axiom system is **complete** if every valid formula of the logic is a theorems.

Proof system

The following axiom system is sound and complete for propositional logic:

Proof system

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 $(\varphi \to (\psi \to \varphi)).$

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Proof system

The following axiom system is sound and complete for propositional logic:

 $\begin{array}{l} \bullet \quad (\varphi \rightarrow (\psi \rightarrow \varphi)). \\ \\ \bullet \quad ((\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))). \end{array} \end{array}$

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Proof system

The following axiom system is sound and complete for propositional logic:

(
$$\varphi \rightarrow (\psi \rightarrow \varphi)$$
).
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($(\neg \varphi \rightarrow \neg \psi) \rightarrow (\psi \rightarrow \varphi)$).

Proof system

The following axiom system is sound and complete for propositional logic:

- $(\varphi \to (\psi \to \varphi)).$
- $((\neg \varphi \to \neg \psi) \to (\psi \to \varphi)).$
- **4** Modus ponens (MP): from φ and $\varphi \to \psi$, infer ψ .

Proof

Example

(http://www.logicinaction.org/)

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Proof

Example

1.
$$p \to ((q \to p) \to p)$$

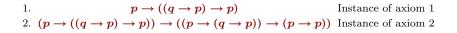
Instance of axiom 1

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Example



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Example

1.
$$p \rightarrow ((q \rightarrow p) \rightarrow p)$$
Instance of axiom 12. $(p \rightarrow ((q \rightarrow p) \rightarrow p)) \rightarrow ((p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p))$ Instance of axiom 23. $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p)$ MP from steps 1 and 2

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Example

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$$p \rightarrow ((q \rightarrow p) \rightarrow p)$$
Instance of axiom 12. $(p \rightarrow ((q \rightarrow p) \rightarrow p)) \rightarrow ((p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p))$ Instance of axiom 23. $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p)$ MP from steps 1 and 24. $(p \rightarrow (q \rightarrow p))$ Instance of axiom 1

Example

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$$p \rightarrow ((q \rightarrow p) \rightarrow p)$$
Instance of axiom 12. $(p \rightarrow ((q \rightarrow p) \rightarrow p)) \rightarrow ((p \rightarrow (q \rightarrow p))) \rightarrow (p \rightarrow p))$ Instance of axiom 23. $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p)$ MP from steps 1 and 24. $(p \rightarrow (q \rightarrow p))$ Instance of axiom 15. $p \rightarrow p$ MP from steps 4 and 3

Example

1.
$$p \rightarrow ((q \rightarrow p) \rightarrow p)$$
Instance of axiom 12. $(p \rightarrow ((q \rightarrow p) \rightarrow p)) \rightarrow ((p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p))$ Instance of axiom 23. $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p)$ MP from steps 1 and 24. $(p \rightarrow (q \rightarrow p))$ Instance of axiom 15. $p \rightarrow p$ MP from steps 4 and 3

Hence, $p \rightarrow p$ is a theorem.

Example



Mrs White is guilty.	w
Miss Scarlet is guilty.	\boldsymbol{s}
Colonel Mustard is guilty.	\boldsymbol{m}

(http://www.logicinaction.org/)

Example



Mrs White is guilty.	w
Miss Scarlet is guilty.	\boldsymbol{s}
Colonel Mustard is guilty.	\boldsymbol{m}

- ▶ At least one of them is guilty.
- ▶ Not all of them are guilty.
- ▶ If Mrs White is guilty, then Colonel Mustard helped her.
- ▶ If Miss Scarlet is innocent then so is Colonel Mustard.

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Example



Mrs White is guilty.	w
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 $egin{aligned} & w ee s ee m \ & \neg(w \wedge s \wedge m) \ & w
ightarrow m \end{aligned}$

Define

$$\Phi := \{w \lor s \lor m, \neg (w \land s \land m), w
ightarrow m, \neg s
ightarrow \neg m\}$$

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Define

$$\Phi := \{w \lor s \lor m, \neg (w \land s \land m), w \to m, \neg s \to \neg m\}$$



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• $\Phi \models s$? • $\Phi \models \neg w$? • $\Phi \models m$?

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Which are the possibilities?

(http://www.logicinaction.org/)

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Remove cases where the **first** premise $w \lor s \lor m$ is false:

Which are the possibilities?

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Remove cases where the **first** premise $w \lor s \lor m$ is false:

Remove cases where the second premise $\neg(w \land s \land m)$ is false:

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Which are the possibilities?

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Remove cases where the **first** premise $w \lor s \lor m$ is false:

 $\begin{array}{c} \left. \left. \left\{ w \right\} \quad \left\{ s \right\} \quad \left\{ m \right\} \quad \left\{ w, s \right\} \quad \left\{ w, m \right\} \quad \left\{ s, m \right\} \quad \left\{ w, s, m \right\} \\ \end{array} \right. \\ \text{Remove cases where the second premise } \neg (w \land s \land m) \text{ is false:} \end{array}$

 $\{ egin{array}{cccc} egin{array}{cccc} egin{array}{cccc} egin{array}{cccc} egin{array}{cccc} w,s & \{w,s\} & \{w,m\} & \{w,s,m\} & \{w,s,$

Which are the possibilities?

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Remove cases where the **first** premise $w \lor s \lor m$ is false:

 $\begin{array}{c} \left. \left. \left. \left\{ w \right\} \right\} \left\{ s \right\} \right\} \left\{ w, s \right\} \left\{ w, m \right\} \left\{ s, m \right\} \left\{ w, s, m \right\} \\ \text{Remove cases where the second premise } \neg (w \land s \land m) \text{ is false:} \\ \left. \left. \left. \left. \left\{ w \right\} \right\} \left\{ s \right\} \left\{ m \right\} \left\{ w, s \right\} \left\{ w, m \right\} \left\{ s, m \right\} \left\{ w, s, m \right\} \right\} \\ \end{array}$

Remove cases where the **third** premise $w \to m$ is false:

Which are the possibilities?

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Remove cases where the **first** premise $w \lor s \lor m$ is false:

 $\begin{array}{c} \left\{ \begin{array}{c} \end{array}\right\} \quad \left\{ w\right\} \quad \left\{ w,s\right\} \quad \left\{ w,m\right\} \quad \left\{ s,m\right\} \quad \left\{ w,s,m\right\} \\ \text{Remove cases where the second premise } \neg (w \land s \land m) \text{ is false:} \\ \left\{ \begin{array}{c} \end{array}\right\} \quad \left\{ w\right\} \quad \left\{ s\right\} \quad \left\{ m\right\} \quad \left\{ w,s\right\} \quad \left\{ w,m\right\} \quad \left\{ s,m\right\} \quad \left\{ w,s,m\right\} \end{array}$

Remove cases where the **third** premise $w \to m$ is false:

Which are the possibilities?

 $\left\{
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ight\} \quad \left\{ w, s
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Remove cases where the **first** premise $w \lor s \lor m$ is false:

Remove cases where the **third** premise $w \to m$ is false:

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Which are the possibilities?

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 $\{ \} \quad \{w\} \quad \{s\} \quad \{m\} \quad \{w,s\} \quad \{w,m\} \quad \{s,m\} \quad \{w,s,m\}$

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Only the following possibilities make **all** premises in Φ true:

(http://www.logicinaction.org/)

Only the following possibilities make **all** premises in Φ true:

• $\Phi \models s$?

(http://www.logicinaction.org/)

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• $\Phi \models s$? Yes!

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Only the following possibilities make **all** premises in Φ true:

• $\Phi \models s$? Yes! • $\Phi \models \neg w$?

Only the following possibilities make **all** premises in Φ true:

- $\Phi \models s$? Yes!
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Only the following possibilities make all premises in Φ true:

- $\Phi \models s$? Yes!
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Updates

Only the following possibilities make all premises in Φ true:

 $\{ \} \quad \{w\} \quad \{s\} \quad \{m\} \quad \{w,s\} \quad \{w,m\} \quad \{s,m\} \quad \{w,s,m\}$

- $\Phi \models s$? Yes!
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- $\Phi \models m$? No!

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Only the following possibilities make all premises in Φ true:

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- $\Phi \models s$? Yes!
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- $\Phi \models \neg m$? No!

Decide whether the following formulas are logically equivalent:

(http://www.logicinaction.org/)

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Decide whether the following formulas are logically equivalent:

• $\varphi \wedge \psi$ and $\neg (\neg \varphi \vee \neg \psi)$.

Decide whether the following formulas are logically equivalent:

- $\varphi \wedge \psi$ and $\neg (\neg \varphi \vee \neg \psi)$.
- $\varphi \to \psi$ and $\neg \varphi \lor \psi$.

Decide whether the following formulas are logically equivalent:

- $\varphi \wedge \psi$ and $\neg (\neg \varphi \vee \neg \psi)$.
- $\varphi \to \psi$ and $\neg \varphi \lor \psi$.
- $\varphi \leftrightarrow \psi$ and $(\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)$.

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Decide whether the following formulas are logically equivalent:

- $\varphi \wedge \psi$ and $\neg (\neg \varphi \lor \neg \psi)$.
- $\varphi \to \psi$ and $\neg \varphi \lor \psi$.
- $\varphi \leftrightarrow \psi$ and $(\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)$.
- What does this tell us?

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Decide whether the following formulas are logically equivalent:

- $\varphi \wedge \psi$ and $\neg (\neg \varphi \vee \neg \psi)$.
- $\varphi \to \psi$ and $\neg \varphi \lor \psi$.
- $\varphi \leftrightarrow \psi$ and $(\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)$.
- What does this tell us?
- Can you find other set of operators strong enough to define the rest of them?

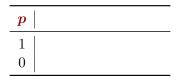
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Consider a single atomic proposition p.

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Consider a single atomic proposition p.



2

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Consider a single atomic proposition p.

p	$arphi_1$	
1	1	
0	1	

2

Consider a single atomic proposition p.

\boldsymbol{p}	$arphi_1$	$arphi_2$	
1	1	1	
0	1	0	

(http://www.logicinaction.org/)

2

Consider a single atomic proposition p.

\boldsymbol{p}	$arphi_1$	$arphi_2$	$arphi_3$	
1	1	1	0	
0	1	0	1	

Consider a single atomic proposition p.

p	$arphi_1$	$arphi_2$	$arphi_3$	$arphi_4$
1	1 1	1	0	0
0	1	0	1	0

(http://www.logicinaction.org/)

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Consider a single atomic proposition p.

p	$arphi_1$	$arphi_2$	$arphi_3$	$arphi_4$
1	1	1	0	0
0	1	0	1	0

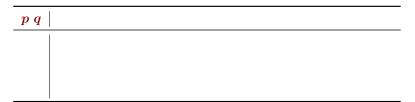
• Can we define φ_1 , φ_2 , φ_3 and φ_4 in our setting?

Consider a single atomic proposition p.

p	$arphi_1$	$arphi_2$	$arphi_3$	$arphi_4$
1 0	1	1	0	0
0	1	0	1	0

- Can we define φ_1 , φ_2 , φ_3 and φ_4 in our setting?
- Can we define each φ_i by using only p and our five connectives \neg , \land , \lor , \rightarrow and \leftrightarrow ?

Consider two atomic propositions p and q.



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Consider two atomic propositions p and q.

p q			
11			
1 0			
$0 \ 1$			
0 0			

Consider two atomic propositions p and q.

$p \ q$	$arphi_1$	$arphi_2$	$arphi_3$	$arphi_4$	$arphi_5$	$arphi_6$	$arphi_7$	$arphi_8$	$arphi_9$	$arphi_{10}$	$arphi_{11}$	$arphi_{12}$	$arphi_{13}$	$arphi_{14}$	$arphi_{15}$	$arphi_{16}$
$\begin{array}{c}1&1\\1&0\\0&1\end{array}$	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1 0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
$0 \ 1$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
0 0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

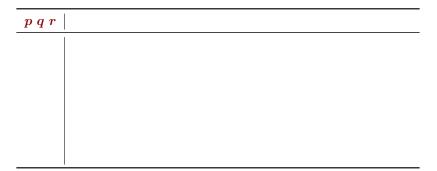
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Consider two atomic propositions p and q.

p q	$ \varphi_1$	$arphi_2$	$arphi_3$	$arphi_4$	$arphi_5$	$arphi_6$	φ_7	$arphi_8$	$arphi_9$	$arphi_{10}$	$arphi_{11}$	$arphi_{12}$	$arphi_{13}$	$arphi_{14}$	$arphi_{15}$	$arphi_{16}$
1 1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
$1 \ 0$	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
$0 \ 1$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
$egin{array}{cccc} 1 & 1 \ 1 & 0 \ 0 & 1 \ 0 & 0 \end{array}$	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

• Can we define each φ_i by using only p, q and our five connectives $\neg, \land, \lor, \rightarrow$ and \leftrightarrow ?

Consider three atomic propositions p, q and r.



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Consider three atomic propositions p, q and r.

p q r		
111		
$1 \ 1 \ 0$		
$1 \ 0 \ 1$		
$1 \ 0 \ 0$		
$0\ 1\ 1$		
$0\ 1\ 0$		
$0 \ 0 \ 1$		
0 0 0		

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Consider three atomic propositions p, q and r.

p q r	
111	
$1 \ 1 \ 0$	
$1 \ 0 \ 1$	
$1 \ 0 \ 0$	
$0\ 1\ 1$	• • •
$0 \ 1 \ 0$	
$0 \ 0 \ 1$	
0 0 0	

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Consider three atomic propositions p, q and r.

p q r	
111	
$1 \ 1 \ 0$	
$1 \ 0 \ 1$	
$1 \ 0 \ 0$	
$0\ 1\ 1$	• • •
$0 \ 1 \ 0$	
$0 \ 0 \ 1$	
0 0 0	

Can we define each φ_i by using only p, q, r and our five connectives ¬, ∧, ∨, → and ↔?